MIDDLE SCHOOL PRESENTATION

BY

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Tasks for this presentation were taken from various resources from the National Council of Supervisors of Mathematics (NCSM)

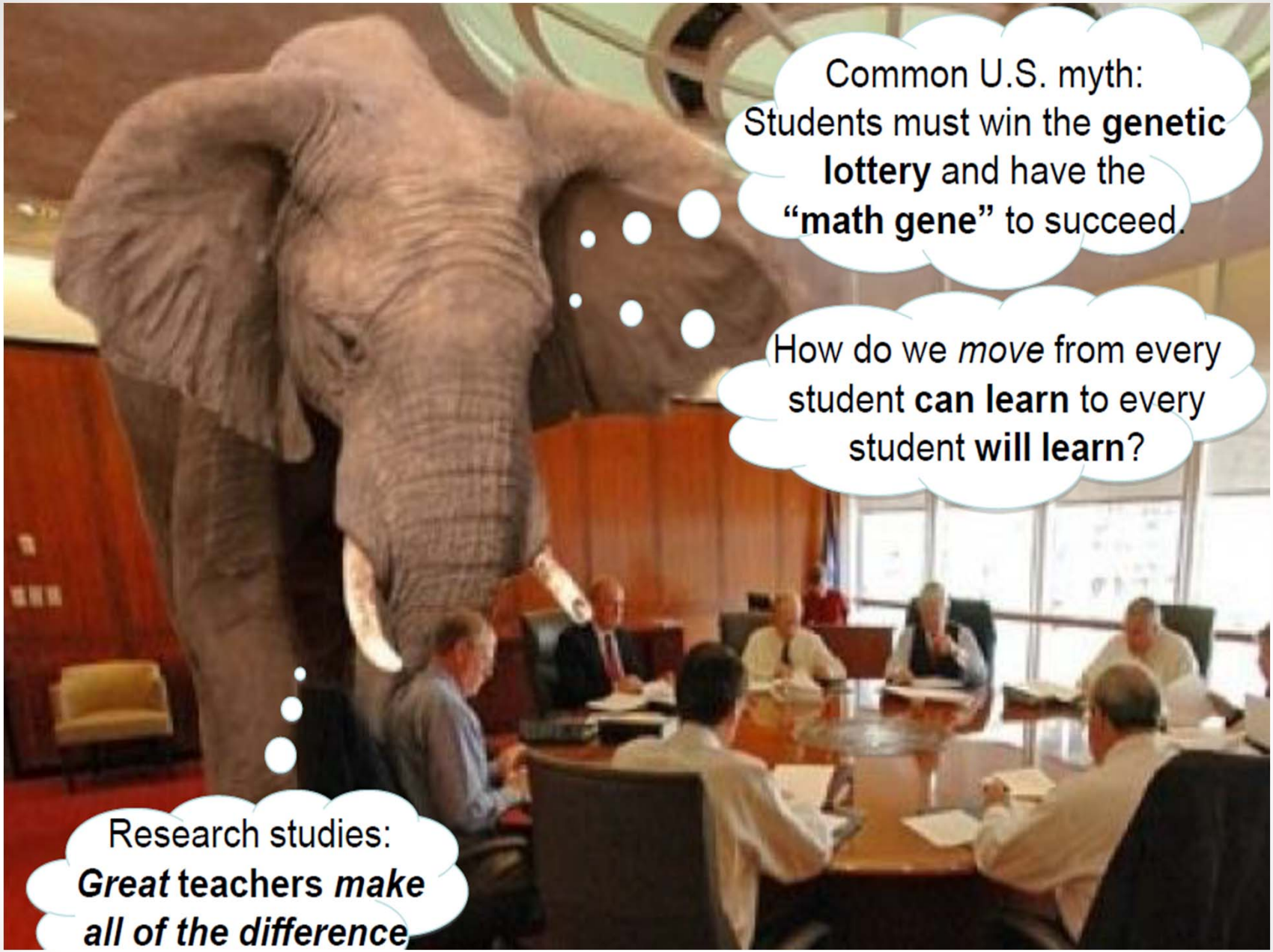
Today's Goals

- To explore the Mathematical Practice Standards
- To consider how the Mathematical Practice Standards are likely to impact your mathematics program and to plan next steps

In particular, participants will

- Have opportunities to examine different mathematical tasks

Video Clip found at
http://www.cgcs.org/Page/1


Common U.S. myth:
Students must win the **genetic lottery** and have the **"math gene"** to succeed.
How do we *move* from every student **can learn** to every student **will learn**?
Research studies:
***Great* teachers *make* all of the difference**

COMMON CORE STATE STANDARDS
FOR MATHEMATICS

STANDARDS FOR MATHEMATICAL PRACTICE

The eight Standards for Mathematical Practice "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years."

STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Content define what students should understand and be able to do in their study of mathematics, balancing procedure and understanding. They stress conceptual understanding of key ideas and continually return to organizing principals. They are organized by grade level for K-8 and by conceptual categories at the high school level.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

DOMAINS AND CONCEPTUAL CATEGORIES		
K-5 Domains	**6-8 Domains**	**HS Conceptual Categories**
Counting and Cardinality (K only)	Ratios and Proportional Relationships (6-7)	Number and Quantity
Operations and Algebraic Thinking (K-5)	The Number System (6-8)	Algebra
Number and Operations in Base Ten (K-5)	Expressions and Equations (6-8)	Functions
Number and Operations – Fractions (3-5)	Functions (8 only)	Modeling
Measurement and Data (K-5)	Geometry (6-8)	Geometry
Geometry (K-5)	Statistics and Probability (6-8)	Statistics and Probability

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"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education." (CCSS, 2010)

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The Standards for Mathematical Practice

Take a moment to examine the first three words of each of the 8 mathematical practices… what do you notice?

Mathematically Proficient Students…

The Standards for [Student] Mathematical Practice

What are the *verbs* that illustrate the student actions for the mathematical practices?

Circle, highlight or underline them for each practice…

What jumps out at you?

The Standards for [Student] Mathematical Practice

SMP1: Explain, make conjectures, persevere…

SMP2: Decontextualize and contextualize …

SMP3: Understand, present and critique…

SMP4: Represent, apply and interpret…

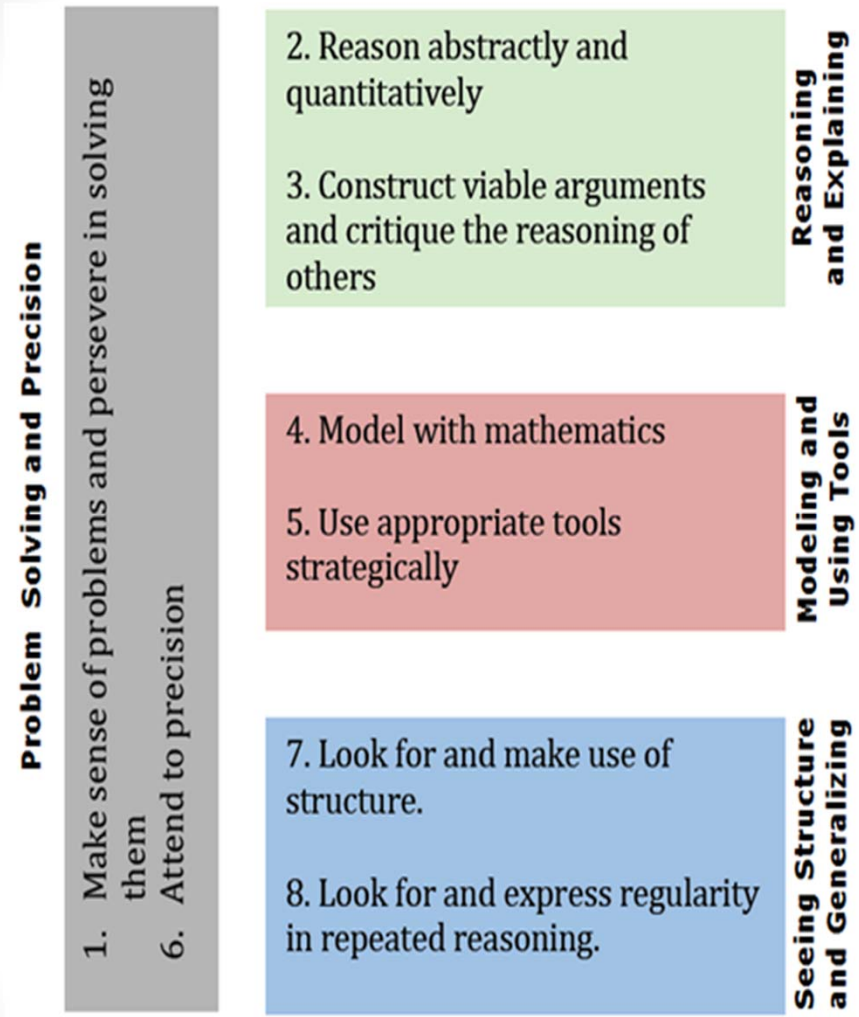
SMP5: Consider, select…

SMP6: Communicate precisely…

SMP7: Discern, recognize and use…

SMP8: Notice, pay attention to and generalize…

Mathematical Practices



- Learned by students when they are engaged in problem solving
- Teachers can highlight specific practices so that students have specific knowledge of a practice
- Tasks can be analyzed both by their content standard as well as their mathematical practice
- The Depth of Knowledge Matrix is used to determine the level of math task

Instructional Issues

Grade 6-8 Cluster verbs…

Write and Interpret…

Understand…

Apply and Extend…

Represent and Interpret

Classify…

GRADES K - 12

TEACHER INSTRUCTIONAL STRATEGIES TO ENGAGE STUDENTS IN THE COMMON CORE "HABITS OF MIND"

Please Note: *As you begin to design learning environments that provide students opportunities to develop the Common Core Mathematical Practice Standards, "Habits of Mind", consider this chart as a quick reference to what the "Habits of Mind" might look like in proficient students. Remember, mathematically literate students can read, write, speak, and listen as a mathematician across all academic content areas.*

MATHEMATICS "HABITS OF MIND"	MATHEMATICALLY PROFICIENT STUDENTS, CAN…
I Make Sense of Problems	• I explain the problem to myself and restate it in my own words • I think about past problems to help determine how to solve a problem in different ways • I use concrete objects or pictures to help conceptualize and solve the problem
I Critique the Reasoning Of Others **I Construct Viable Arguments**	• I understand and use stated assumptions, definitions, and previously established results in constructing my arguments • I make conjectures and build a logical progression of statements to explore the truth of my conjectures • I analyze situations by breaking them into cases, and can recognize and use counter examples • I justify my conclusions, communicate clearly to others, and respond to the arguments of others
I Attend to Precision	• I communicate precisely to others, using clear definitions in discussion with others • I state the meaning of the symbols I choose, and carefully specify and label units of measure • I calculate accurately and efficiently
I Persevere in Solving Problems	• I analyze, make conjectures, and consider analogous problems to gain insight into a variety of possible solution strategies • I monitor and evaluate my own progress and change course if necessary • I check my answers, explain correspondences between my mathematical representations, and ask myself; "Does this make sense?"
EMPOWERMENT *Empowerment is the process of increasing the capacity of individuals or groups to make choices and to transform those choices into desired actions and outcomes. Central to this process are actions which both build individual and collective assets, "Habits of Mind".*	
I Use Appropriate Tools Strategically	• I consider the available tools when solving a mathematical problem • I make sound decisions about when each tool might be helpful, recognizing both the insight to be gained and their limitations • I use technological tools to explore and deepen my understanding, and enable me to visualize and compare • I can identify relevant online mathematical resources, and use to pose or solve problems
I Model with Mathematics	• I apply the math I know to solve problems arising in everyday life • I am comfortable making assumptions and approximations to simplify a complicated situation • I identify important quantities in a practical situation and use tools to map their relationships • I analyze mathematical relationships to draw conclusions • I interpret my results in the context of the situation and reflect on whether the results make sense, and improve the model if it has not served its purpose

I Reason Abstractly and Quantitatively	• I make sense of quantities and their relationships in problem situations
	• I translate given information to create a mathematical representation for a concept, bring two complementary abilities to bear on problems involving quantitative relationships
	  ○ I represent my mathematically thinking symbolically
	  ○ I demonstrate understanding of the meaning of the symbols involved
	• I create a coherent representation of the problem at hand

I Look For and Express Regularity in Repeated Reasoning	• I notice if calculations are repeated, and look for both general methods and shortcuts • I maintain oversight of my solution process, while attending to the details • I continually evaluate the reasonableness of my results
I Look For and Make Use Of Structure	• I look closely to discern a pattern or structure • I can step back for an overview of a mathematical equation or expression, and shift my perspective • I can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects

Standards for Mathematical Practice in a Classroom

McDonald's Claim

Wikipedia reports that 8% of all Americans eat at McDonalds every day.

310 million Americans and 12,800 McDonalds…

Do you believe the Wikipedia report to be true? Create a mathematical argument to justify your position.

McDonald's Claim Problem

- Which mathematical *practices* are needed to complete the task?
- What mathematics *content* is needed to complete the task?

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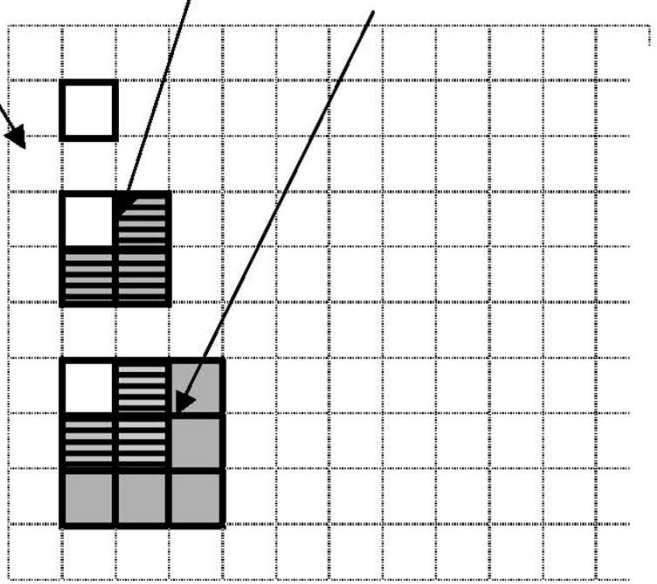
Kate makes a pattern of squares.

She starts with 1 square, then adds 3 more, then 5 more, and so on.



1 x 1 square

2 x 2 square

3 x 3 square

1. Draw the next shape in Kate's pattern.
2. How many new squares did you add?
3. What size square did you make?

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1. Write down the numbers of squares that make the number pattern. 1, 4, ___, ____, ____

2. Write the next two lines of the number pattern.

1 = 1 x 1 = 1

1 + 3 = 2 x 2 = 4

_____

_____

3. Use the number pattern to find the total of the following numbers:

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

Show your work.

4. Write down the number pattern that gives a total of 169. Explain your work.

6–° ¸ˇ1–#$

5. Use the number pattern to total the numbers.

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 100

Show your work.

1 25 81
3 11 +19
5 13 100
7 15
+9 +17
25 81

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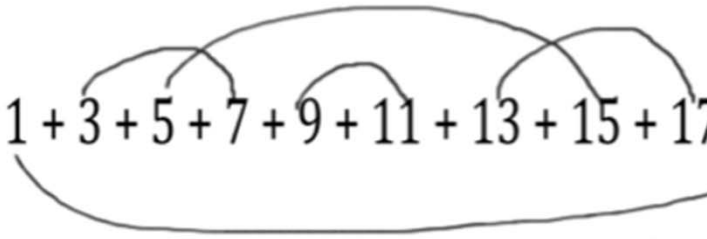
5. Use the number pattern to total the numbers.



1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 100

Show your work.

1+19 = 20
3+7 = 10
5+15 = 20
9+11 = 20
13+17 = 30
100

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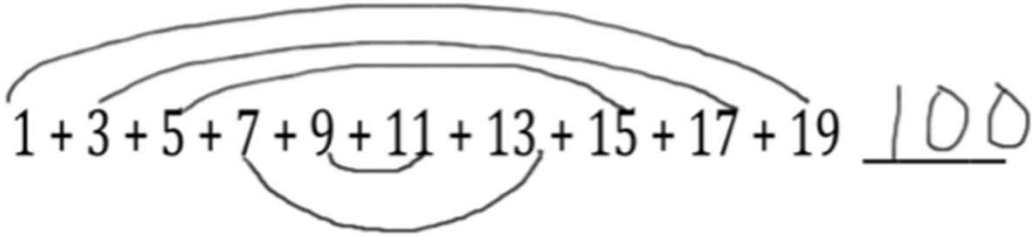
5. Use the number pattern to total the numbers.



1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 100

Show your work.

20
20
20
20
20
100

6–° ˛ˇ1–#&

5. Use the number pattern to total the numbers.

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 _____

Show your work.

1 = 1 x 1 = 1
1+3 = 2 x 2 = 4
etc.
1+3+5+7+9 = 5x5 = 25
1+3+5+7+9+11 = 6x6 = 36
etc.
1+3+5+7+9+11+13+15 = 8x8 = 64
1+3+5+7+9+11+13+15+17 = 9x9 = 81
1+3+5+7+9+11+13+15+17+19 = 10x10
= 100

1
+3
4
+5
9
+7
16
+9
25
+11
36
+13
49
+15
64
+17
81
+19
100

Comparing Students A,H,G and C

- *What evidence might support the conclusion that these students are using structure?*

- *What evidence might support the conclusion that these students are using regularity in repeated reasoning?*

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6. Write down the number pattern that gives a total of 169.

Explain your work.

1+3+5+7+9+11+13+15+17+19+21+23+25
Because 1+3+5+7+9+11+13+15+17+19=100
and there are 10 numbers in all. If I added the next three odd numbers, there will be 13 numbers and 13×13=169.

6–° ¸ˇ1–#&#

6. Write down the number pattern that gives a total of 169.

Explain your work.

1+3+5+7+9+11+13+15+17+19+21+23+25
= 13x13 = 169.
I started with 100 and added on 21, then 23, and then 25. Then it came up with 169.

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- *Compare the explanations given by these two students.*
- *How is each using regularity in repeated reasoning?*
- *Which student seems to be closer to making a generalization? What is your evidence?*

Grade 7 Mathematics Sample ER Item

MAT.07.ER.3.000EE.D.156

Sample Item ID:	MAT.07.ER.3.000EE.D.156
Grade:	07
Primary Claim:	**Claim 3: Communicating Reasoning** Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Expressions and Equations
Secondary Content Domain(s):	The Number System
Assessment Target(s):	3 A: Test propositions or conjectures with specific examples. 1 D: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
Standard(s):	7.EE.3, 7.EE.1, 7.NS.2
Mathematical Practice(s):	1, 3, 5
DOK:	3
Item Type:	ER
Score Points:	3
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	Part of PT set; some of the task can be AI scored.

Renee, Susan, and Martha will share the cost to rent a vacation house for a week.

- Renee will pay 40% of the cost.
- Susan will pay 0.35 of the cost.
- Martha will pay the remainder of the cost.

Part A

Martha thinks that she will pay $\frac{1}{3}$ of the cost. Is Martha correct?

Use mathematics to justify your answer.

Part B

The cost to rent a vacation house for a week is $850. How much will Renee, Susan, and Martha each pay to rent this house for a week?

Renee will pay $ ☐

Susan will pay $ ☐

Martha will pay $ ☐

Scoring Rubric:

Responses to this item will receive 0–3 points, based on the following:

3 points: The student shows a thorough understanding of how solving a real-world problem can lead to flawed reasoning. The student shows a thorough understanding of solving a real-world problem involving numeric expressions with rational numbers. This is shown by the student indicating that Martha is incorrect with justification and correctly calculates the amount each person pays.

2 points: The student shows good understanding of solving a real-world problem involving numeric expressions with rational numbers. The student recognizes that Martha will not pay 1/3 of the cost, but makes a minor error in showing the calculations to support the response to Part A. This error is then consistently applied in responding to Part B.

1 point: The student shows limited understanding of solving a real-world problem involving numeric expressions with rational numbers. The student can correctly calculate Renee's and/or Susan's portion of the trip, but other parts of the response are incorrect.

0 points: The student shows inconsistent or no understanding of how solving a real-world problem can lead to flawed reasoning or how to solve a real-world problem involving numeric expressions with rational numbers. Simply claiming that Martha's portion of the trip rental is not equal to 1/3 is not sufficient to earn any points.

Sample Top-Score Response:

Part A

Martha is incorrect. She will pay 1/4 of the cost.

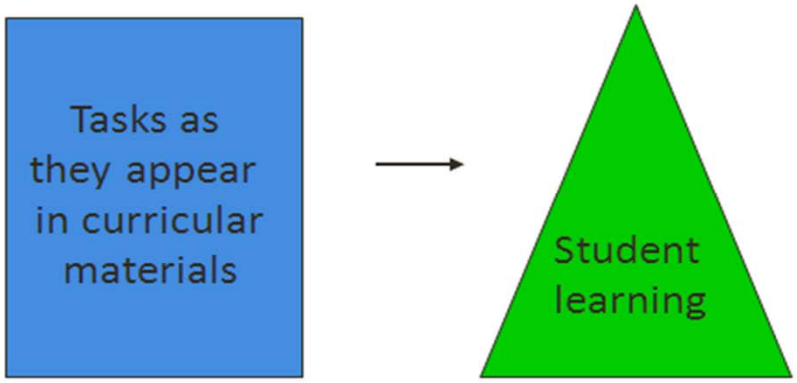
$$1-(40\%+0.35)$$
$$1-(0.40+0.35)$$
$$1-0.75$$
$$0.25=\frac{25}{100}=\frac{1}{4}$$

Part B

Renee - $340
Susan - $297.50
Martha - $212.50

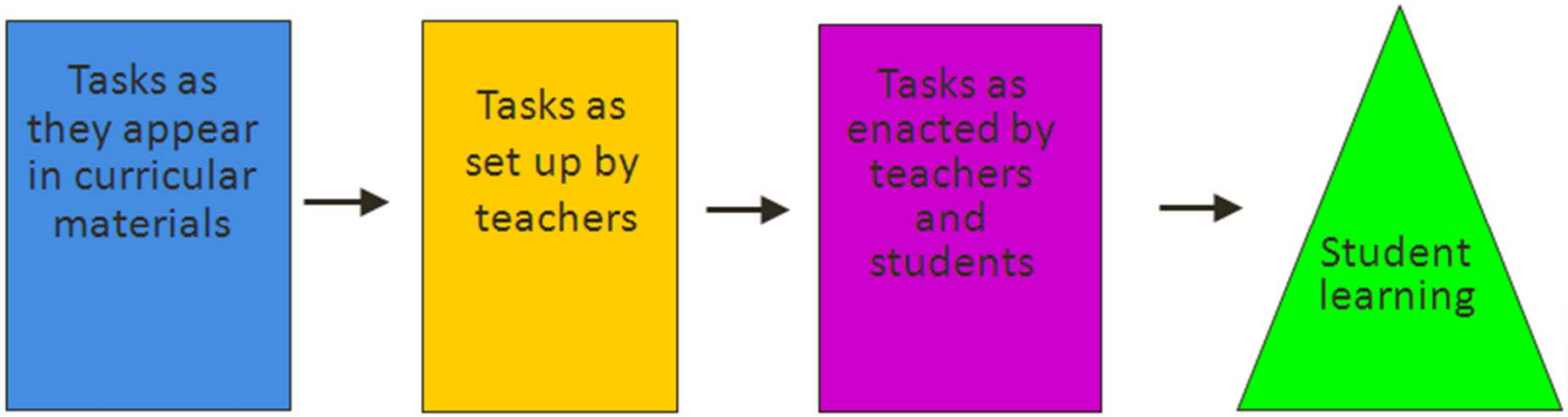
The Nature of Tasks Used in the Classroom …

Will Impact Student Learning!



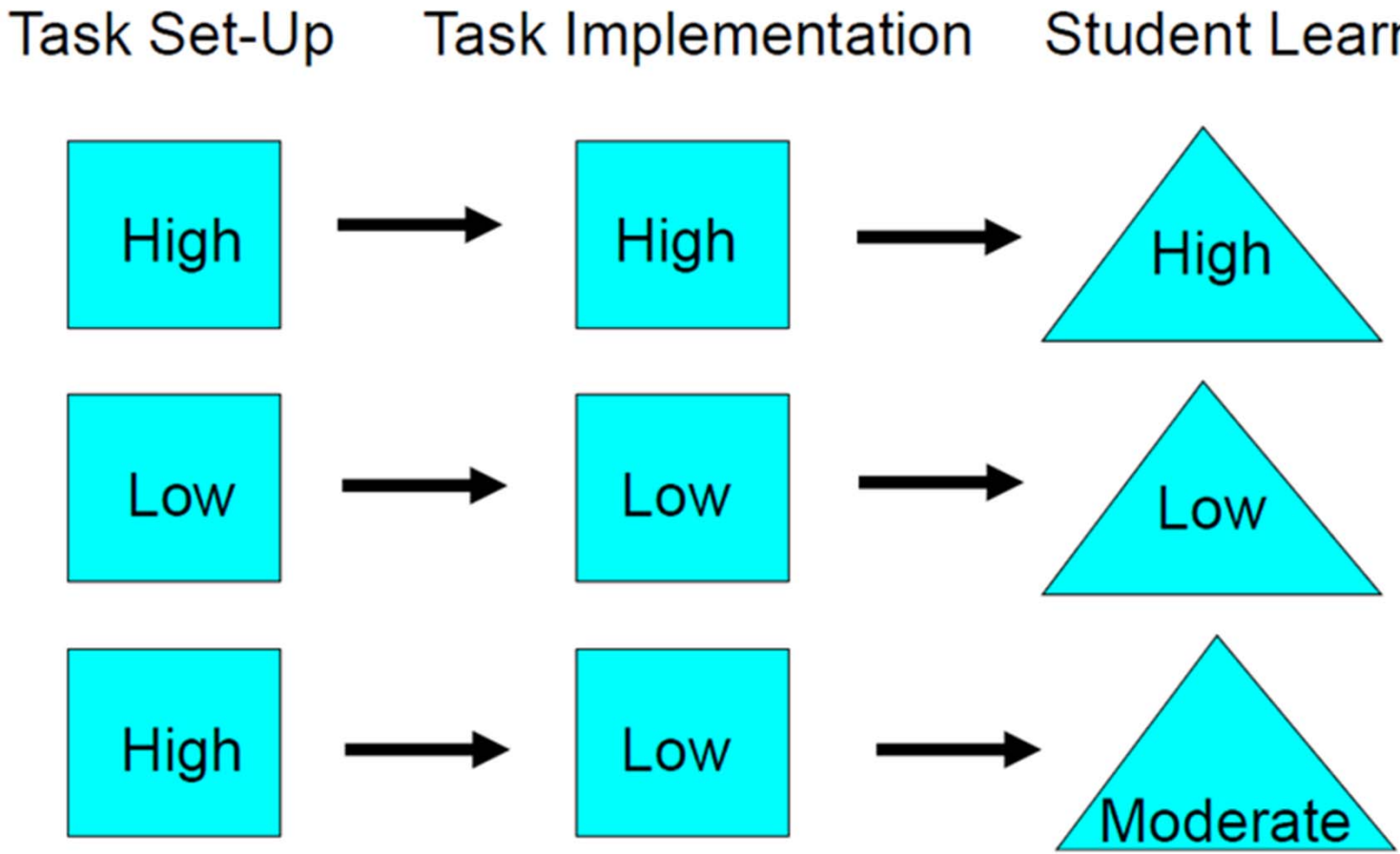
But, WHAT TEACHERS DO with the tasks matters too!

The Mathematical Tasks Framework



Stein, Grover & Henningsen (1996)
Smith & Stein (1998)
Stein, Smith, Henningsen & Silver (2000)

Maintaining High Cognitive Demand Matters!



Stein & Lane, 1996

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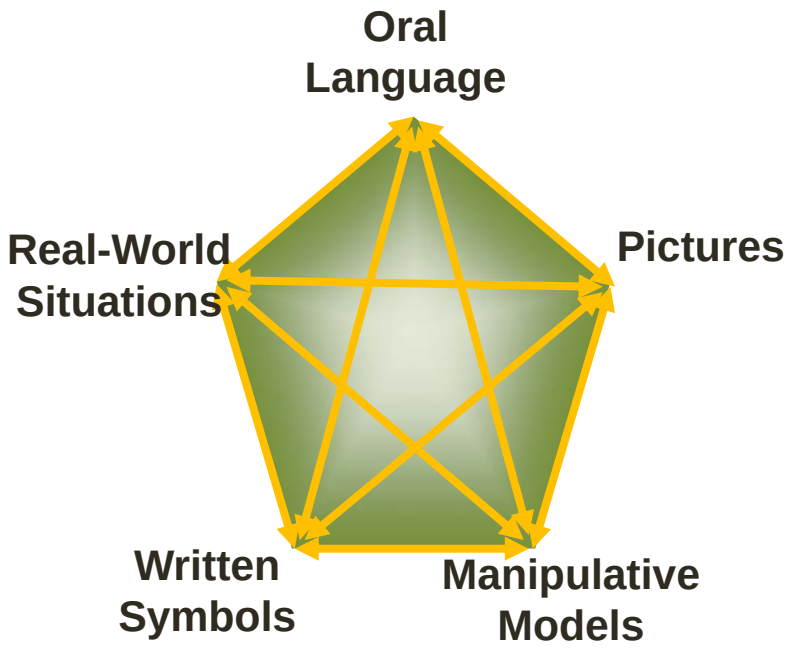
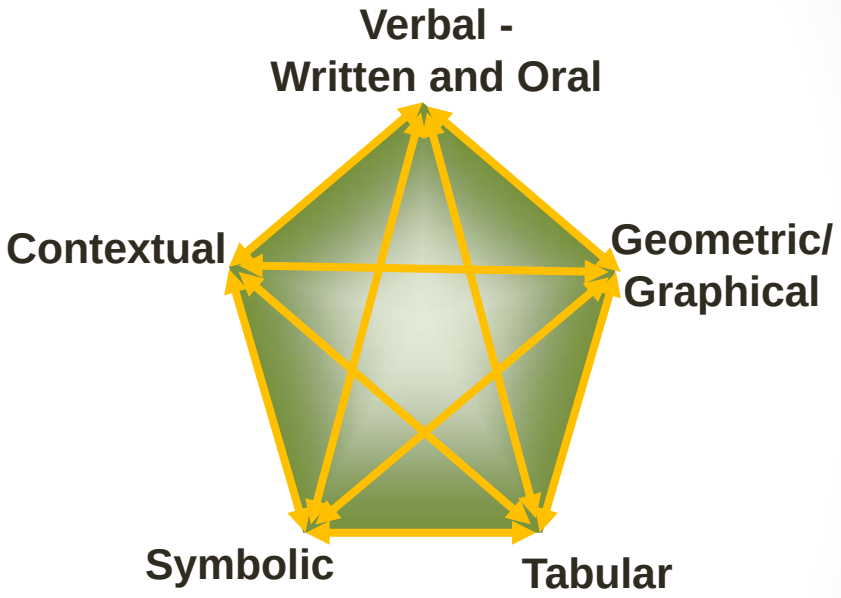


Elementary



Secondary

Adapted from Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations among Representations in Mathematics Learning and Problem Solving. In C. Janvier, (Ed.), Problems of Representations in the Teaching and Learning of Mathematics (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

The Standards for Mathematical Practice Require

Lesson Planning from the
"Student's Point of View"

Deepening the student learning Experience…

POWERFUL ACTIONS

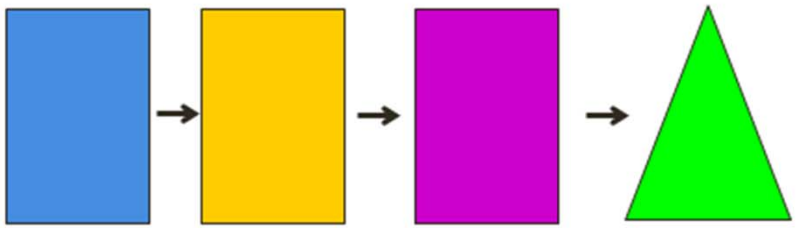
1. Apply the Standards for Mathematical Practice as student practices
2. Attend to procedural fluency
3. Maintain high cognitive demand tasks
4. Solve problems in a variety of ways

POWERFUL ACTIONS

5. **Shift to "unit lesson design" from the student's point of view**
6. **Make mathematics visible –Use the real world**
7. **Use formative assessment practices daily and hourly to assess student learning and use enhanced assessment tools when possible**

Planning to Support Students' Opportunity to Learn

Select a typical task (or a related set of problems) from your instructional materials and design a lesson so that it offers more opportunities for students to develop both the *content* and *practice* standards.



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Resources

Balanced Assessment	balancedassessments.concord.org
California Common Core Website	http://www.cde.ca.gov/re/cc/
Common Core Tools	commoncoretools.worldpress.com
Council of the Great City Schools	http://www.cgcs.org/site/default.aspx?PageID=1
Engage New York	http://engageny.org/
Illustrative Math	illustrativemathematics.org
Inside Mathematics	insidemathematics.org/index.php/home
Mathematics Assessment Project	map.mathshell.org/materials/index.php
NCSM Great Tasks	mathedleadership.org/member/docs/ccss/GreatTasksHandout2012.pdf
NCTM	illuminations.nctm.org
	www.nctm.org/smtasks/
PARCC	www.parconline.org
Smarter Balanced Assessments (Released Questions)	http://www.smarterbalanced.org/news/smarter-balanced-releases-sample-assessment-items-and-performance-tasks/
Tools for the Common Core	http://commoncoretools.me/author/wgmccallum/